Übungen zur Vorlesung "Feldtheorie"

SoSe 2011

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Präsenzübung Freitag 15.07.

Problem 1 The electroweak interaction has the gauge group $U(1)\times SU(2)$. Correspondingly, there are two coupling constants $g^{U(1)} = g_1$ and $g^{SU(2)} = g_2$, and two Lie-algebra valued gauge fields $\mathcal{A}_\mu^{U(1)} = A_\mu^0 X_0$ and $\mathcal{A}_\mu^{SU(2)} = \sum_{a=1}^3 A_\mu^a X_a$, where we choose the bases $X_0 = -i$ and $X_a = -\frac{i}{2}\sigma^a$ of the respective Lie algebras. The physically normalized vector potentials are $B_\mu^0 = \hbar/g_1 \cdot A_\mu^0$ and $B_\mu^a = \hbar/g_2 \cdot A_\mu^a$.

The Higgs doublet $\underline{\Phi}$ transforms in the identical representation. Its covariant derivative is therefore $D_\mu\Phi = \partial_\mu\Phi - \mathcal{A}^{U(1)}\Phi - \mathcal{A}^{SU(2)}\Phi$.

a) Compute the contribution arising from the Lagrangean term

$$\frac{1}{2}(D_\mu\Phi)^+ D^\mu\Phi,$$

that is quadratic in the gauge potentials ("mass term"), for $\Phi_0 = \binom{0}{v}$.

b) Find the orthogonal change of basis $(B^0, B^3) \to (A, Z)$ such that the mass term has the form

$$\frac{c^2}{2\hbar^2}\Big(M_W^2 \cdot W_\mu^+ W^{-\mu} + M_Z^2 \cdot Z_\mu Z^\mu\Big),$$

where $W^\pm = B^1 \pm iB^2$, and compute the mass ratio M_Z/M_W.

c) Compute the Dirac couplings for Dirac doublets $\underline{\psi} = \binom{\psi_+}{\psi_-}$:

$$-g_1 j_{D,0}^\mu B_\mu^0 - g_2 \sum_a j_{D,a}^\mu B_\mu^a$$

where $j_{D,0}^\mu = i\underline{\psi}^+\gamma^0\gamma^\mu \pi_Y(X_0)\underline{\psi}$ and $j_{D,a}^\mu = i\underline{\psi}^+\gamma^0\gamma^\mu X^a\underline{\psi}$, and π_Y is the "hypercharge Y" representation of the Lie algebra $u(1)$ given by $\pi_Y(X_0) = -iY\mathbf{1}_2$. Select the contribution involving the massless field A_μ (the photon field), and express it in terms of the Dirac currents $j_{D,\pm}^\mu = \psi_\pm^+\gamma^0\gamma^\mu\psi_\pm$ for the "up" and "down" components of the Dirac doublet:

$$-j_{\text{elm}}^\mu A_\mu = -(q_+ j_{D,+}^\mu + q_- j_{D,-}^\mu)A_\mu,$$

and thus read off the electric charges $q_\pm$. Express them in terms of the coupling constants, the hypercharge, and the "isospin" $T_3 = \pm\frac{1}{2}$ for $\psi_\pm$.