Übungen zur Vorlesung "Feldtheorie"

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Problem 1 *The FLRW metric*

a) Write the Minkowski metric $ds^2 = c^2dt^2 - d\vec{x}^2$ in spatial polar coordinates, and show that it is a special case of the Friedmann-Lemaître-Robertson-Walker metric

$$ds^2 = c^2dt^2 - A(t)^2\Big[\frac{dr^2}{1-er^2} + r^2\big(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\big)\Big].$$

b) The FLRW metric can also be written as

$$ds^2 = a(\tau)^2\Big[c^2d\tau^2 - \frac{dr^2}{1-er^2} + r^2\big(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\big)\Big].$$

Find the relation between the FLRW time t and the "conformal time" τ, and between the functions $A(t)$ and $a(\tau)$.

Problem 2 *Vacuum Cosmology*

In an expanding universe, the matter content will be more and more "diluted", and will become negligible at very late times.

a) Consider the vacuum ($\varepsilon = p = 0$) Friedmann equations

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2 + ec^2}{A^2} = \Lambda c^2, \qquad 3\frac{\dot{A}^2 + ec^2}{A^2} = \Lambda c^2,$$

for $e = +1$. Show that the cosmological constant must be positive in this case, and solve the equations for $A(t)$.

b) Show that the resulting metric is that of deSitter spacetime. The latter is the 4-dimensional "surface" $(X^0)^2 - (X^1)^2 - (X^2)^2 - (X^3)^2 - (X^4)^2 = -L^2$ embedded into an (auxiliary) 5-dimensional Minkowski spacetime.

Hint: parametrize $\underline{X}$ by 4-dimensional coordinates $\underline{X} = (X^0, \sqrt{L^2 + (X^0)^2} \cdot E)$, where $E = (\cos\chi, \sin\chi\cos\vartheta, \sin\chi\sin\vartheta\cos\varphi, \sin\chi\sin\vartheta\sin\varphi)$ is a 4-dimensional unit vector in polar coordinates. The deSitter metric is $(dX^0)^2 - (dX^1)^2 - (dX^2)^2 - (dX^3)^2 - (dX^4)^2$ expressed in terms of the coordinates $X^0, \chi, \vartheta, \varphi$. Then adjust the FLRW coordinates t and r to match the deSitter metric.

c) Visualize deSitter spacetime by suppressing two of its spatial dimensions.