

Übungen zur Vorlesung ‘Feldtheorie’

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Problem 1 *The Riemann tensor and its symmetries*

Consider the Riemann tensor given by

$$R_{\sigma\mu\nu}^{\rho} = \partial_{\mu}\Gamma_{\nu\sigma}^{\rho} - \partial_{\nu}\Gamma_{\mu\sigma}^{\rho} + \Gamma_{\mu\alpha}^{\rho}\Gamma_{\nu\sigma}^{\alpha} - \Gamma_{\nu\alpha}^{\rho}\Gamma_{\mu\sigma}^{\alpha}$$

Show that the following identities are fulfilled

- a) $R_{\sigma\mu\nu}^{\rho} = -R_{\sigma\nu\mu}^{\rho}$,
- b) $R_{\sigma\mu\nu}^{\rho} + R_{\mu\nu\sigma}^{\rho} + R_{\nu\sigma\mu}^{\rho} = 0$,
- c) $R_{\rho\sigma\mu\nu} = -R_{\sigma\rho\mu\nu}$, in which $R_{\rho\sigma\mu\nu} = g_{\rho\lambda}R_{\sigma\mu\nu}^{\lambda}$,
- d) $R_{\rho\sigma\mu\nu} = R_{\mu\nu\rho\sigma}$.

Show that, in dimension d , a tensor with the above symmetries possess

$$DoF(R_{\sigma\mu\nu}^{\rho}) = \frac{1}{12}d^2(d^2 - 1)$$

independent components. Consider in particular $d = 2$ and $d = 3$ and show that the Riemann tensor can be expressed completely in term of the Ricci tensor $R_{\mu\nu}$ and the scalar curvature R .

Finally, show that the field strength $F_{\mu\nu}$ verify the equation

$$D_{\alpha}F_{\beta\gamma} + D_{\beta}F_{\gamma\alpha} + D_{\gamma}F_{\alpha\beta} = 0$$

if $F_{\mu\nu} = D_{\mu}A_{\nu} - D_{\nu}A_{\mu}$ thanks to one of the identity of the Riemann tensor.

Problem 2 *More identities*

Prove the following identities:

$$\begin{aligned} \partial_{\gamma}g_{\alpha\beta} &= \Gamma_{\alpha\beta\gamma} + \Gamma_{\beta\alpha\gamma}, & g_{\alpha\mu}\partial_{\gamma}g^{\mu\beta} &= -g^{\mu\beta}\partial_{\gamma}g_{\alpha\mu}, \\ D_{\alpha}A^{\alpha} &= \frac{1}{\sqrt{|g|}}\partial_{\alpha}(\sqrt{|g|}A^{\alpha}), & g^{\mu\nu}D_{\mu}D_{\nu}\phi &= \frac{1}{\sqrt{|g|}}\partial_{\mu}\sqrt{|g|}g^{\mu\nu}\partial_{\nu}\phi, \end{aligned}$$

with $g = \det g_{\mu\nu}$.

Problem 3 For the following actions find the Euler-Lagrange equations

$$\begin{aligned} S_{\phi} &= \frac{1}{2} \int [g^{\rho\sigma}(D_{\rho}\phi)(D_{\sigma}\phi) - m^2\phi^2]\sqrt{|g|}dx \\ S_A &= \frac{-1}{4} \int g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\sqrt{|g|}dx \end{aligned}$$

with $F_{\mu\nu} = D_{\mu}A_{\nu} - D_{\nu}A_{\mu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$ (check it!) and