Übungen zur Vorlesung “Feldtheorie”

SoSe 2011

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Präsenzübung Freitag 27.05.

Problem 1 Addition of velocities.

Consider a system (call it A) moving at velocity $\vec{v}$ with respect to your (inertial) frame of reference. Consider a second system (call it B) moving at the velocity $\vec{w}$ with respect to the system A. Show that then B appears to move with the velocity

$$\vec{u} = \frac{\vec{v} + \vec{w}_{\|} + \gamma^{-1}(\vec{v})\vec{w}_{\perp}}{1 + \frac{\vec{v}.\vec{w}}{c^2}}$$

with $\vec{w}$ the component of $\vec{w}$ along $\vec{v}$, resp. $\vec{w}_{\perp}$ perpendicular to $\vec{v}$.

Problem 2 On the Lorentz group

The Lorentz matrices Λ are those leaving the quadratic form

$$(x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2$$

invariant, with $x^0 = ct$. Show that

$$\Lambda^0{}_0 \geq 1 \quad \text{or} \quad \Lambda^0{}_0 \leq 1$$

and

$$\det\Lambda = +1 \quad \text{or} \quad \det\Lambda = -1$$

Introduce the time and space inversions

$$T = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}, \quad P = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

to conclude that any such transformation is obtained by a composition of T, P and $\{\Lambda | \Lambda^0{}_0 \geq 1, \det\Lambda = +1\}$.

Problem 3 Relativistic motion of a free point particle.

Consider the lagrangian of a free massive point particle:

$$L = -m_0 c^2 \sqrt{1 - \frac{|\dot{\vec{x}}(t)|^2}{c^2}}.$$

a) Derive its equation of motion.

b) Show that for low velocities (wrt c) one recovers the usual non-relativistic lagrangian.