Übungen zur Vorlesung “Feldtheorie”

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Problem 1 The dielectric sphere

Consider a sphere of radius R and dielectric constant ε set in an uniform electrostatic field $\vec{E}(\vec{x})$ such as

$$\lim_{|\vec{x}|\to\infty} \vec{E}(\vec{x}) = E_0\vec{e}_z.$$

a) Solve the Laplace equation on the potential $\phi(\vec{x})$ inside and outside the sphere in spherical coordinates after noticing the rotational symmetry around the z-axis.
Hint. Remember that Legendre polynomials $P_l(x)$ are solutions of $(1-x^2)f'' - 2xf' + l(l+1)f = 0$. Or, simply, $Y_{l0} \propto P_l$ (*cf.* previous exercises sheet).

b) Apply the boundary conditions

$$\lim_{|\vec{x}|\nearrow R} \vec{E}_{\text{tan}}(\vec{x}) = \lim_{|\vec{x}|\searrow R} \vec{E}_{\text{tan}}(\vec{x}),$$

$$\lim_{|\vec{x}|\nearrow R} \varepsilon\vec{E}_{\text{norm}}(\vec{x}) = \lim_{|\vec{x}|\searrow R} \vec{E}_{\text{norm}}(\vec{x}),$$

with “tan” (resp. “norm”) the tangential (resp. normal) component with respect to the sphere, to further reduce the expression of the potential $\phi(\vec{x})$.

c) Write the electric field inside and outside the sphere. Consider in particular $\vec{E}(\vec{x}) - E_0\vec{e}_z$ to find the dipole moment induced by the dielectric sphere.

Problem 2 Are the Maxwell equations invariant under time inversion and under space reflections? How should the fields and the sources transform under these symmetries?