Übungen zur Vorlesung “Feldtheorie”

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Problem 1 Quadrupoles

Consider a charge distribution with rotational symmetry around an axis (assumed to be the z axis, to simplify).

a) Show that the 2^l-pole parts of the electrostatic potential

$$\phi_l(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\frac{1}{l!}\sum m_{i_1\dots i_l}\partial_{i_1}\dots\partial_{i_l}\frac{1}{|\vec{x}|}$$

are multiples of $r^{-l-1}Y_{lm}(\vartheta,\varphi)$ with $m=0$ only.

b) The same holds for the gravitational potential of a rotational symmetric mass distribution. The sun is not an exact sphere, but an ellipsoid due to centrifugal forces. Show that $\phi_l = 0$ for all odd l.

c) Substitute the ellipsoid by a sphere of mass M, plus a pair of “negative point masses” $-\mu$ at the poles, plus a ring of mass 2μ along the equator. Compute the quadrupole moment

$$m_{ij} = \int x_i x_j\,\rho(\vec{x})\,d^3x.$$

d) Consider planets on circular orbits in the equatorial plane. Derive the relation between the radius and the period of the orbit due to the quadrupole moment, and discuss the deviation from Kepler’s 3rd law.

Problem 2 Screening (“Faraday’s cage”)

Show that the electrostatic field inside an arbitrarily shaped closed metallic container does not depend on the charges outside the container. Show also that the converse is not true, i.e., the field outside does depend on the charges inside.