Übungen zur Vorlesung "Feldtheorie"

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Problem 1 Consider the Sine-Gordon Lagrangean density (in one space dimension)

$$\mathcal{L} = \frac{1}{2}\dot{\xi}^2 - \frac{c^2}{2}(\partial_x \xi)^2 + \mu^2\Big(\cos(\beta\xi) - 1\Big)$$

for the field $\xi(x,t)$, with constant model parameters c, β, and μ.

a) Write down the equation of motion, and show that

$$\xi(x,t) = \frac{4}{\beta}\arctan e^{\gamma(x-x_0-vt)\cdot\beta\mu/c}$$

is a solution for arbitrary x_0 and v with $|v| < c$, provided γ is properly chosen. Determine γ! What are the asymptotic values $\xi(x = \pm\infty, t)$?

b) Prove the following "particle-like" features of this solution:

(i) Its energy density ρ_E and momentum density ρ_P are localized around the trajectory $x(t) = x_0 + vt$, with a constant width.

(ii) The total energy $E = \int \rho_E \, dx$ and total momentum $P = \int \rho_P \, dx$ satisfy the relation of a relativistic particle $E^2 = M_0^2 c^4 + P^2 c^2$. Determine the rest mass M_0.

Hints for (a) and (b): Derive and use the formulae

$$\dot{\xi} = -v\partial_x\xi = -2(v/c)\gamma\mu\sin(\beta\xi/2)$$

and

$$\sin(\beta\xi/2) = 1/\cosh\big(\gamma(x - x_0 - vt)\cdot\beta\mu/c\big).$$