Übungen zur Vorlesung “Feldtheorie”

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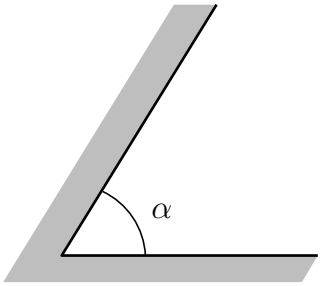
Problem 1 Let $d = 2$ and consider an incompressible frictionless fluid circulating inside a wedge with opening angle $\alpha < \pi$.



a) Find the most general stationary curl-free velocity field $\vec{v}(x, y)$ with the boundary condition that along the walls $\vec{v}$ is parallel to the walls.
Hint: represent $\vec{v} = \vec{\nabla}\Psi$ and determine Ψ by making the separation ansatz, in polar coordinates, $\Psi(r, \theta) = f(r)g(\theta)$.

b) Discard solutions for which $|\vec{v}|$ grows as $r \to \infty$ and select the solution with the slowest decay as $r \to \infty$. How is the behavior of $\vec{v}(r, 0)$ along the wall?

Problem 2 Determine the orbits for the linear planar velocity fields

$$v_i(\vec{x}) = \sum_{j=1}^{2} A_{ij} x_j$$

for

$$a)\ A = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \qquad b)\ A = \begin{pmatrix} -a & 0 \\ 0 & -a \end{pmatrix}, \qquad c)\ A = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix},$$

$$d)\ A = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}, \qquad e)\ A = \begin{pmatrix} 0 & a \\ a & 0 \end{pmatrix}$$

and discuss the distortion of a fluid volume that was originally a square $-\ell \le x \le \ell$, $-\ell \le y \le \ell$.